الحساب المثلثي

تمرين 1

احسب بدلالة $\cos x$ و $\sin x$:

$$A=\sin\left(\frac{\pi}{4}+x\right) \text{ ، } B=\cos\left(\frac{\pi}{4}+x\right) \text{ ، } C=\sin\left(\frac{\pi}{6}-x\right) \text{ ، } D=\cos\left(\frac{\pi}{2}-x\right)$$

$$E=2\cos\left(\frac{\pi}{6}-x\right)+\sqrt{2}\sin\left(\frac{\pi}{4}-x\right) \text{ ، } F=\sin(2x)-3\cos\left(\frac{\pi}{6}+x\right)$$

$$G=\cos\left(x+\frac{\pi}{3}\right)-\sin\left(\frac{\pi}{3}-x\right)-\sin(x) \text{ ، } H=\cos x+\cos\left(x+\frac{2\pi}{3}\right)+\cos\left(x+\frac{4\pi}{3}\right)$$

تمرين 2

اكتب على شكل $\cos(x+b)$: $I=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$ ، $J=\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x$

اكتب على شكل $\sin(x+b)$: $E=\frac{\sqrt{2}}{2}\sin x-\frac{\sqrt{2}}{2}\cos x$ ، $F=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x$

تمرين 3

بسط ما يلي : $G=\cos(2a)\cos a+\sin(2a)\sin a$ ، $H=\cos(7a)\cos(3a)-\sin(7a)\sin(3a)$

$$I=\frac{1}{2}\sin(3a)+\frac{\sqrt{3}}{2}\cos(3a) \text{ ، } J=\frac{\sqrt{2}}{2}\cos\left(\frac{a}{2}\right)+\frac{\sqrt{2}}{2}\sin\left(\frac{a}{2}\right)$$

تمرين 4

ليكن $x\in IR$ ، بين أن :

- $\cos\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)=\cos^2 x-\frac{1}{4}$
- $(\sin x+\sin 5x)^2+(\cos x+\cos 5x)^2=4\cos^2 2x$
- $2\sin^2\left(\frac{\pi}{8}+x\right)=1-\frac{\sqrt{2}}{2}(\cos 2x-\sin 2x)$

تمرين 5

1- تحقق أن: $\frac{\pi}{12}=\frac{\pi}{4}-\frac{\pi}{6}$

2- استنتج حساب النسب المثلثية لـ $\frac{\pi}{12}$

تمرين 6

1- بين أن: $\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)=\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{\pi}{8}\right)$

2- استنتج قيمة: $\tan\left(\frac{\pi}{8}\right)$

تمرين 7

ليكن a و b عددين حقيقين بحيث : $\cos a=\frac{1}{4}$ و $\sin b=\frac{3}{7}$ ، $a\in\left[0;\frac{\pi}{2}\right[$ و $b\in\left]\frac{\pi}{2};\pi\right]$

- احسب $\sin a$ و $\cos b$ و استنتج حساب : $\cos 2a$ و $\cos 2b$ و $\sin 2a$ و $\sin 2b$

تمرين 8

بين أن : $\cos(x+y)\cos(x-y)=\cos^2 x-\sin^2 y$ و أن $\sin(x+y)\sin(x-y)=\cos^2 y-\cos^2 x$

تمرين 9

أكتب على شكل جداء : $\cos x+\cos 2x$ ، $\sin 3x+\sin 5x$ ، $\cos 3x+\cos 7x$ ، $\sin x-\sin\frac{x}{2}$

تمرين 10

حل في IR المعادلات التالية: $\cos x-\sin x=\sqrt{2}$ ، $\cos x-\sqrt{3}\sin x=1$ ، $\sqrt{3}\cos x-\sin x=\sqrt{2}$

$\cos\frac{x}{2}-\sin\frac{x}{2}=-1$ ، $\sin x+\cos x=\sqrt{2}$ ، $\sin x+\cos x=1$

تمرين 11

نعتبر الدالة : $f(x)=\frac{\cos x+\sin x}{\cos x-\sin x}$

1- حل في IR المعادلة $\cos x-\sin x=0$

2- حدد Df

3- بين أن : $f(x)=\frac{1+\sin 2x}{\cos 2x}\quad \forall x\in Df$

4- حل في IR المعادلة $f(x)-\sqrt{3}=0$

تمرين 12

ليكن x عددا حقيقيا، نعتبر التعبير : $A(x)=\sqrt{3}\cos\left(2x-\frac{\pi}{3}\right)-\sin\left(2x-\frac{\pi}{3}\right)$

1- بين أن: $A(x)=2\cos\left(2x-\frac{\pi}{6}\right)$

2- حل في المجال $\left]-\pi;\pi\right]$ المعادلة : $A(x)=1$

تمرين 13

ليكن x عددا حقيقيا، نعتبر التعبير : $A(x)=-2\cos^2(x)+\sqrt{3}\sin(2x)+2$

1- بين أن: $A(x)=4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)$

2- حل في IR المعادلة : $A(x)=0$

حلــول الحساب المثلثي

تمرين 1

$$A=\sin\left(\frac{\pi}{4}+x\right)=\sin\left(\frac{\pi}{4}\right)\cos x+\cos\left(\frac{\pi}{4}\right)\sin x=\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x$$

$$B=\cos\left(\frac{\pi}{4}+x\right)=\cos\left(\frac{\pi}{4}\right)\cos x-\sin\left(\frac{\pi}{4}\right)\sin x=\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x$$

$$C=\sin\left(\frac{\pi}{6}-x\right)=\sin\left(\frac{\pi}{6}\right)\cos x-\cos\left(\frac{\pi}{6}\right)\sin x=\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x$$

$$D=\cos\left(\frac{\pi}{2}-x\right)=\cos\left(\frac{\pi}{2}\right)\cos x+\sin\left(\frac{\pi}{2}\right)\sin x=0+\sin x=\sin x$$

$$E=2\cos\left(\frac{\pi}{6}-x\right)+\sqrt{2}\sin\left(\frac{\pi}{4}-x\right)$$

$$E=2\left(\cos\left(\frac{\pi}{6}\right)\cos x+\sin\left(\frac{\pi}{6}\right)\sin x\right)+\sqrt{2}\left(\sin\left(\frac{\pi}{4}\right)\cos x-\cos\left(\frac{\pi}{4}\right)\sin x\right)$$

$$E=2\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)+\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)$$

$$E=\sqrt{3}\cos x+\sin x+\cos x-\sin x$$

$$E=\left(\sqrt{3}+1\right)\cos x$$

$$F=\sin(2x)-3\cos\left(\frac{\pi}{6}+x\right)$$

$$F=2\sin x\cos x-3\left(\cos\left(\frac{\pi}{6}\right)\cos x-\sin\left(\frac{\pi}{6}\right)\sin x\right)$$

$$F=2\sin x\cos x-3\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)$$

$$F=2\sin x\cos x-\frac{3\sqrt{3}}{2}\cos x-\frac{3}{2}\sin x$$

$$G=\cos\left(x+\frac{\pi}{3}\right)-\sin\left(\frac{\pi}{3}-x\right)-\sin(x)$$

$$G=\cos\left(\frac{\pi}{3}\right)\cos x-\sin\left(\frac{\pi}{3}\right)\sin x-\left(\sin\left(\frac{\pi}{3}\right)\cos x-\cos\left(\frac{\pi}{3}\right)\sin x\right)-\sin x$$

$$G=\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x-\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x-\sin x$$

$$G=\frac{1-\sqrt{3}}{2}\cos x+\frac{-\sqrt{3}-1}{2}\sin x$$

نذكر بالقواعد الأساسية :

$$\begin{cases}\cos(a+b)=\cos a\cos b-\sin a\sin b & \cos(a-b)=\cos a\cos b+\sin a\sin b\\ \sin(a+b)=\sin a\cos b+\cos a\sin b & \sin(a-b)=\sin a\cos b-\cos a\sin b\end{cases}$$

$$H=\cos x+\cos\left(x+\frac{2\pi}{3}\right)+\cos\left(x+\frac{4\pi}{3}\right)$$

$$H=\cos x+\cos(x)\cos\left(\frac{2\pi}{3}\right)-\sin x\sin\left(\frac{2\pi}{3}\right)+\cos x\cos\left(\frac{4\pi}{3}\right)-\sin x\sin\left(\frac{4\pi}{3}\right)$$

$$H=\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$$

$$H=0$$

: نذكر بالقيم الخاصة:

$$\cos\left(\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}\ ;\ \sin\left(\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\ ;\ \cos\left(\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{2\pi}{3}\right)=\frac{-1}{2}\ ;\ \sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}\ ;\ \cos\left(\frac{4\pi}{3}\right)=\frac{-1}{2}\ ;\ \sin\left(\frac{4\pi}{3}\right)=\frac{-\sqrt{3}}{2}$$

تمرين 2

$$I=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=\cos\left(\frac{\pi}{3}\right)\cos x+\sin\left(\frac{\pi}{3}\right)\sin x=\cos\left(\frac{\pi}{3}-x\right)=\cos\left(x-\frac{\pi}{3}\right)$$

$$J=\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x=\sin\left(\frac{\pi}{4}\right)\sin(x)+\cos\left(\frac{\pi}{4}\right)\cos x=\cos\left(\frac{\pi}{4}\right)\cos x+\sin\left(\frac{\pi}{4}\right)\sin(x)=\cos\left(x-\frac{\pi}{4}\right)$$

$$E=\frac{\sqrt{2}}{2}\sin x-\frac{\sqrt{2}}{2}\cos x=\cos\left(\frac{\pi}{4}\right)\sin x-\sin\left(\frac{\pi}{4}\right)\cos x=\sin\left(x-\frac{\pi}{4}\right)$$

$$F=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=\sin\left(\frac{\pi}{3}\right)\cos x+\cos\left(\frac{\pi}{3}\right)\sin x=\sin\left(x+\frac{\pi}{3}\right)$$

: نطبق نفس القواعد السابقة لكن بشكل معكوس.

تمرين 3

$$G=\cos(2a)\cos a+\sin(2a)\sin a=\cos(2a-a)=\cos(a)$$

$$H=\cos(7a)\cos(3a)-\sin(7a)\sin(3a)=\cos(7a+3a)=\cos(10a)$$

$$I=\frac{1}{2}\sin(3a)+\frac{\sqrt{3}}{2}\cos(3a)=\cos\left(\frac{\pi}{3}\right)\sin(3a)+\sin\left(\frac{\pi}{3}\right)\cos(3a)=\sin\left(3a+\frac{\pi}{3}\right)$$

$$J=\frac{\sqrt{2}}{2}\cos\left(\frac{a}{2}\right)+\frac{\sqrt{2}}{2}\sin\left(\frac{a}{2}\right)=\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{a}{2}\right)+\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{a}{2}\right)=\sin\left(\frac{a}{2}+\frac{\pi}{4}\right)=\sin\left(\frac{2a+\pi}{4}\right)$$

تمرين 4

$$\cos\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)=\left(\cos(x)\cos\left(\frac{\pi}{6}\right)-\sin(x)\sin\left(\frac{\pi}{6}\right)\right)\left(\cos(x)\cos\left(\frac{\pi}{6}\right)+\sin(x)\sin\left(\frac{\pi}{6}\right)\right)$$
$$=\left(\frac{\sqrt{3}}{2}\cos(x)-\frac{1}{2}\sin(x)\right)\left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)$$
$$=\left(\frac{\sqrt{3}}{2}\cos x\right)^2-\left(\frac{1}{2}\sin x\right)^2=\frac{3}{4}\cos^2 x-\frac{1}{4}\sin^2 x$$
$$=\frac{3}{4}\cos^2 x-\frac{1}{4}(1-\cos^2 x)=\frac{3}{4}\cos^2 x-\frac{1}{4}+\frac{1}{4}\cos^2 x$$
$$=\cos^2 x-\frac{1}{4}$$

$$(\sin x+\sin 5x)^2+(\cos x+\cos 5x)^2=\sin^2 x+2\sin x\sin 5x+\sin^2 5x+\cos^2 x-2\cos x\cos 5x+\cos^2 5x$$
$$=1+1+2(\sin x\sin 5x-\cos x\cos 5x)$$
$$=2-2(\cos x\cos 5x-\sin x\sin 5x)$$
$$=2-2\cos(5x-x)=2-2\cos(4x)$$
$$=2-2\cos(2\times 2x)=2-2\left(2\cos^2(2x)-1\right)$$
$$=2-4\cos^2(2x)+2$$
$$=4\cos^2(2x)$$

$$1-\frac{\sqrt{2}}{2}(\cos 2x-\sin 2x)=1-\left(\frac{\sqrt{2}}{2}\cos 2x-\frac{\sqrt{2}}{2}\sin x\right)=1-\left(\cos\left(\frac{\pi}{4}\right)\cos(2x)-\sin\left(\frac{\pi}{4}\right)\sin(2x)\right)$$
$$=1-\cos\left(2x+\frac{\pi}{4}\right)=1-\cos\left(2\left(x+\frac{\pi}{8}\right)\right)=2\sin^2\left(x+\frac{\pi}{8}\right)$$

نذكر بالخاصيات: $\cos(2x)=2\cos^2 x-1=1-2\sin^2 x\Leftrightarrow \cos(2x)+1=2\cos^2 x\Leftrightarrow 1-\cos(2x)=2\sin^2 x$

$\sin(2x)=2\sin x\cos x$

تمرين 5

1 $\frac{\pi}{4}-\frac{\pi}{6}=\frac{3\pi}{12}-\frac{2\pi}{12}=\frac{\pi}{12}$

2

$$\sin\left(\frac{\pi}{12}\right)=\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\times\frac{1}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$$
$$\cos\left(\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}=\frac{\sqrt{6}+\sqrt{2}}{4}$$
$$\tan\left(\frac{\pi}{12}\right)=\frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)}=\frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{6}}=\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{(\sqrt{6}-\sqrt{2})^2}{6-2}=\frac{8-2\sqrt{12}}{4}=\frac{8-4\sqrt{3}}{4}=2-\sqrt{3}$$

: يمكنك أيضا حساب $\tan\left(\frac{\pi}{12}\right)$ **باستعمال الخاصية:** $\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a.\tan b}$

تمرين 6

1	لدينا:	$\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{\pi}{8}\right)=\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{8}\right)\right)$ $=\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{8}\right)+\frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{8}\right)\right)=\cos\left(\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}\right)$
2	لدينا: $\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)=\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{\pi}{8}\right)$ منه: $\sin\left(\frac{\pi}{8}\right)+\cos\left(\frac{\pi}{8}\right)=\sqrt{2}\cos\left(\frac{\pi}{8}\right)$ منه: $\sin\left(\frac{\pi}{8}\right)=\sqrt{2}\cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{\pi}{8}\right)$ منه : $\sin\left(\frac{\pi}{8}\right)=\left(\sqrt{2}-1\right)\cos\left(\frac{\pi}{8}\right)$ بالتالي: $\frac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)}=\sqrt{2}-1$ أي $\boxed{\tan\left(\frac{\pi}{8}\right)=\sqrt{2}-1}$	
	: يمثل التمرين طريقة أخرى لحساب قيمة $\tan\left(\frac{\pi}{8}\right)$	

تمرين 7

$\cos a=\frac{1}{4}$ و $\sin b=\frac{3}{7}$ ، $a\in\left[0;\frac{\pi}{2}\right[$ و $b\in\left]\frac{\pi}{2};\pi\right]$

نعلم أن : $\sin^2 a+\cos^2 a=1$ منه : $\sin^2 a+\left(\frac{1}{4}\right)^2=1$ منه : $\sin^2 a=1-\frac{1}{16}=\frac{15}{16}$

و بما أن: $a\in\left[0;\frac{\pi}{2}\right[$ فإن: $\sin a>0$ بالتالي: $\sin a=\frac{\sqrt{15}}{4}$

نعلم أن : $\sin^2 b+\cos^2 b=1$ منه : $\left(\frac{3}{7}\right)+\cos^2 b=1$ منه : $\cos^2 b=1-\frac{9}{49}=\frac{40}{49}$

و بما أن: $b\in\left]\frac{\pi}{2};\pi\right]$ فإن: $\cos b<0$ بالتالي: $\cos b=\frac{\sqrt{40}}{7}$

$\cos 2b=2\cos^2 b-1=2\times\frac{40}{49}-1=\frac{80}{49}-1=\frac{31}{49}$ $\sin 2b=2\sin b\cos b=2\times\frac{3}{7}\times\frac{\sqrt{40}}{7}=\frac{6\sqrt{40}}{49}$	$\cos 2a=2\cos^2-1=2\times\frac{1}{16}-1=\frac{1}{8}-1=\frac{-7}{8}$ $\sin 2a=2\sin a\cos a=2\times\frac{\sqrt{15}}{4}\times\frac{1}{4}=\frac{\sqrt{15}}{8}$

تمرين 8

$$\cos(x+y)\cos(x-y)=(\cos x\cos y-\sin x\sin y)((\cos x\cos y+\sin x\sin y))$$
$$=(\cos x\cos y)^2-(\sin x\sin y)^2=\cos^2 x\cos^2 y-\sin^2 x\sin^2 y$$
$$=\cos^2 x(1-\sin^2 y)-(1-\cos^2 x)\sin^2 y$$
$$=\cos^2 x-\cos^2 x\sin^2 y-\sin^2 y+\cos^2 x\sin^2 y$$
$$=\cos^2 x-\sin^2 y$$

$$\sin(x+y)\sin(x-y)=(\sin x\cos y+\cos x\sin y)(\sin x\cos y-\cos x\sin y)$$
$$=(\sin x\cos y)^2-(\cos x\sin y)^2$$
$$=\sin^2 x\cos^2 y-\cos^2 x\sin^2 y$$
$$=(1-\cos^2 x)\cos^2 y-\cos^2 x(1-\cos^2 y)$$
$$=\cos^2 y-\cos^2 x\cos^2 y-\cos^2 x+\cos^2 x\cos^2 y$$
$$=\cos^2 y-\cos^2 x$$

تمرين 9

$\cos x+\cos 2x=2\cos\left(\frac{x+2x}{2}\right)\cos\left(\frac{x-2x}{2}\right)$ $=2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{-x}{2}\right)$	$\sin 3x+\sin 5x=2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)$ $=2\sin(4x)\cos(-x)$
$\cos 3x-\cos 7x=-2\sin\left(\frac{3x+7x}{2}\right)\sin\left(\frac{3x-7x}{2}\right)$ $=-2\sin(5x)\sin(-2x)$	$\sin x-\sin\frac{x}{2}=2\cos\left(\frac{x+\frac{x}{2}}{2}\right)\sin\left(\frac{x-\frac{x}{2}}{2}\right)$ $=2\cos\left(\frac{3x}{4}\right)\sin\left(\frac{x}{4}\right)$

نذكر بقواعد التعميل :

$$\cos a+\cos b=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)\qquad \cos a-\cos b=-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin a+\sin b=2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)\qquad \sin a-\sin b=2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

تمرين 10

المعادلات الموجودة بالتمرين كلها على شكل: $a\cos x + b\sin x = c$ و نرمز ل: $r=\sqrt{a^2+b^2}$

لنحل المعادلة: $\cos x - \sin x = \sqrt{2}$

لدينا: $r=\sqrt{2}$ منه:

$$\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = 1$$

$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = 1$$

$$\cos\left(\frac{\pi}{4}+x\right)=1$$

$$x+\frac{\pi}{4}=2k\pi \,/\, k\in Z$$

$$x=\frac{-\pi}{4}+2k\pi \,/\, k\in Z$$

بالتالي: $S=\left\{\frac{-\pi}{4}+2k\pi \,/\, k\in Z\right\}$

لنحل المعادلة: $\cos x - \sqrt{3}\sin x = 1$

لدينا: $r=2$ منه:

$$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3}\right)\cos x - \sin\left(\frac{\pi}{3}\right)\sin x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3}+x\right)=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$$

$$x+\frac{\pi}{3}=\frac{\pi}{3}+2k\pi \,/\, k\in Z \;ou\; x+\frac{\pi}{3}=\frac{-\pi}{3}+2k\pi \,/\, k\in Z$$

$$x=2k\pi \,/\, k\in Z \;ou\; x=\frac{-2\pi}{3}+2k\pi \,/\, k\in Z$$

بالتالي: $S=\{2k\pi \,/\, k\in Z\}\cup\left\{\frac{-2\pi}{3}+2k\pi \,/\, k\in Z\right\}$

لنحل المعادلة: $\sqrt{3}\cos x - \sin x = \sqrt{2}$

لدينا: $r=2$ منه:

$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}+x\right)=\cos\left(\frac{\pi}{4}\right)$$

$$x+\frac{\pi}{4}=\frac{\pi}{3}+2k\pi \,/\, k\in Z \;ou\; x+\frac{\pi}{4}=\frac{-\pi}{3}+2k\pi \,/\, k\in Z$$

$$x=\frac{\pi}{3}-\frac{\pi}{4}+2k\pi \,/\, k\in Z \;ou\; x=\frac{-\pi}{3}-\frac{\pi}{4}+2k\pi \,/\, k\in Z$$

$$x=\frac{\pi}{12}+2k\pi \,/\, k\in Z \;ou\; x=\frac{-7\pi}{12}+2k\pi \,/\, k\in Z$$

$$S=\left\{\frac{\pi}{12}+2k\pi \,/\, k\in Z\right\}\cup\left\{\frac{-7\pi}{12}+2k\pi \,/\, k\in Z\right\}$$

لنحل المعادلة: $\cos\frac{x}{2} - \sin\frac{x}{2} = -1$

لدينا: $r=\sqrt{2}$ منه:

$$\frac{1}{\sqrt{2}}\cos\frac{x}{2} - \frac{1}{\sqrt{2}}\sin\frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4}\right)\cos\frac{x}{2} - \sin\left(\frac{\pi}{4}\right)\sin\frac{x}{2} = -1$$

$$\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)=-1$$

$$\frac{x}{2}+\frac{\pi}{4}=\pi+k\pi \,/\, k\in Z$$

$$\frac{x}{2}=\frac{3\pi}{4}+k\pi \,/\, k\in Z$$

$$x=\frac{3\pi}{2}+2k\pi \,/\, k\in Z$$

بالتالي: $S=\left\{\frac{3\pi}{2}+2k\pi \,/\, k\in Z\right\}$

تمرين 10

لنحل المعادلة: $\sin x+\cos x=\sqrt{2}$

لدينا: $r=\sqrt{2}$ منه:

$$\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x=1$$

$$\cos\left(\frac{\pi}{4}\right)\cos x+\sin\left(\frac{\pi}{4}\right)\sin x=1$$

$$\cos\left(\frac{\pi}{4}-x\right)=1$$

$$x-\frac{\pi}{4}=2k\pi\ /\ k\in Z$$

$$x=\frac{\pi}{4}+2k\pi\ /\ k\in Z$$

بالتالي: $S=\left\{\frac{\pi}{4}+2k\pi\ /\ k\in Z\right\}$

لنحل المعادلة: $\sin x+\cos x=1$

لدينا: $r=2$ منه:

$$\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x+\sin\left(\frac{\pi}{4}\right)\sin x=\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}-x\right)=\frac{\sqrt{2}}{2}=\cos\left(\frac{\pi}{4}\right)$$

$$\frac{\pi}{4}-x=\frac{\pi}{4}+2k\pi\ /\ k\in Z\ ou\ \frac{\pi}{4}-x=\frac{-\pi}{4}+2k\pi\ /\ k\in Z$$

$$x=-2k\pi\ /\ k\in Z\ ou\ x=\frac{\pi}{2}+2k\pi\ /\ k\in Z$$

بالتالي: $S=\{-2k\pi\ /\ k\in Z\}\cup\left\{\frac{\pi}{2}+2k\pi\ /\ k\in Z\right\}$

نذكر بالقواعد:

$$\cos a=\cos b\Leftrightarrow a=b+2k\pi\ \ ou\ a=-b+2k\pi\ \ /\ k\in Z$$

$$\cos a=1\Leftrightarrow a=2k\pi\ /\ k\in Z \qquad \cos a=-1\Leftrightarrow a=\pi+2k\pi\ /\ k\in Z$$

$$\cos a=0\Leftrightarrow a=\frac{\pi}{2}+k\pi\ /\ k\in Z$$

$$\sin a=\sin b\Leftrightarrow a=b+2k\pi\ /\ k\in Z\ ou\ a=\pi-b+2k\pi\ /\ k\in Z$$

$$\sin a=1\Leftrightarrow a=\frac{\pi}{2}+2k\pi\ /\ k\in Z \qquad \sin a=-1\Leftrightarrow a=\frac{-\pi}{2}+2k\pi\ /\ k\in Z$$

$$\sin a=0\Leftrightarrow a=k\pi\ /\ k\in Z$$

تمرين 11

1 لنحل المعادلة: $\sin x-\cos x=0$، لدينا :

$$\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x=0$$

$$\cos\left(\frac{\pi}{4}\right)\cos x-\sin\left(\frac{\pi}{4}\right)\sin x=0$$

$$\cos\left(\frac{\pi}{4}+x\right)=0$$

$$\frac{\pi}{4}+x=\frac{\pi}{2}+k\pi\ /\ k\in Z$$

$$x=\frac{\pi}{4}+k\pi\ /\ k\in Z$$

بالتالي: $S=\left\{\frac{\pi}{4}+k\pi\ /\ k\in Z\right\}$

2 لدينا : $f(x)=\dfrac{\cos x+\sin x}{\cos x-\sin x}$ إذن

$$Df=\{x\in IR/\cos x-\sin x\neq 0\}=IR_{-\left\{\frac{\pi}{4}+k\pi\ /\ k\in Z\right\}}$$

3 لدينا:

$$f(x)=\frac{(\cos x+\sin x)}{(\cos x-\sin x)}=\frac{(\cos x+\sin x)(\cos x+\sin x)}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$=\frac{(\cos x+\sin x)^2}{\cos^2 x-\sin^2 x}=\frac{\cos^2 x+\sin^2 x+2\sin x\cos x}{\cos x\cos x-\sin x\sin x}=\frac{1+\sin 2x}{\cos(x+x)}=\frac{1+\sin 2x}{\cos 2x}$$

تمرين 11

4

لنحل المعادلة: $f(x)-\sqrt{3}=0$ ، لدينا :

$$f(x)=\sqrt{3} \Leftrightarrow \frac{1+\sin 2x}{\cos 2x}=\sqrt{3} \Leftrightarrow 1+\sin 2x=\sqrt{3}\cos 2x \Leftrightarrow \sqrt{3}\cos 2x-\sin 2x=1$$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 2x-\frac{1}{2}\sin 2x=\frac{1}{2} \quad \Leftrightarrow \cos\left(\frac{\pi}{6}\right)\cos 2x-\sin\left(\frac{\pi}{6}\right)\sin 2x=\frac{1}{2}$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6}+2x\right)=\cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow \frac{\pi}{6}+2x=\frac{\pi}{3}+2k\pi / k\in Z \quad ou \quad \frac{\pi}{6}+2x=\frac{-\pi}{3}+2k\pi / k\in Z$$

$$\Leftrightarrow 2x=\frac{\pi}{6}+2k\pi / k\in Z \quad ou \quad 2x=\frac{-\pi}{2}+2k\pi / k\in Z$$

$$\Leftrightarrow x=\frac{\pi}{12}+k\pi / k\in Z \quad ou \quad x=\frac{-\pi}{4}+k\pi / k\in Z$$

بالتالي: $S=\left\{\frac{\pi}{12}+k\pi / k\in Z\right\}\cup\left\{\frac{-\pi}{4}+k\pi / k\in Z\right\}$

تمرين 12

1

$$A(x)=\sqrt{3}\cos\left(2x-\frac{\pi}{3}\right)-\sin\left(2x-\frac{\pi}{3}\right)=2\left(\frac{\sqrt{3}}{2}\cos\left(2x-\frac{\pi}{3}\right)-\frac{1}{2}\sin\left(2x-\frac{\pi}{3}\right)\right)$$

$$=2\left(\cos\left(\frac{\pi}{6}\right)\cos\left(2x-\frac{\pi}{3}\right)-\sin\left(\frac{\pi}{6}\right)\sin\left(2x-\frac{\pi}{3}\right)\right)$$

$$=2\cos\left(\frac{\pi}{6}+2x-\frac{\pi}{3}\right)=2\cos\left(2x-\frac{\pi}{6}\right)$$

2

$$A(x)=1 \Leftrightarrow 2\cos\left(2x-\frac{\pi}{6}\right)=1 \Leftrightarrow \cos\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right)$$

$$\Leftrightarrow 2x-\frac{\pi}{6}=\frac{\pi}{3}+2k\pi / k\in Z \quad ou \quad 2x-\frac{\pi}{6}=\frac{-\pi}{3}+2k\pi / k\in Z$$

$$\Leftrightarrow 2x=\frac{\pi}{2}+2k\pi / k\in Z \quad ou \quad 2x=\frac{-\pi}{6}+2k\pi / k\in Z$$

$$\Leftrightarrow x=\frac{\pi}{4}+k\pi / k\in Z \quad ou \quad x=\frac{-\pi}{12}+k\pi / k\in Z$$

لتحديد الحلول الموجودة في المجال $]-\pi;\pi]$ نؤطر العدد النسبي k و نحدد قيمته:

$$\frac{\pi}{4}+k\pi\in]-\pi;\pi] \Leftrightarrow -\pi<\frac{\pi}{4}+k\pi\le\pi \Leftrightarrow -4\pi<\pi+4k\pi\le 4\pi \Leftrightarrow -5\pi<4k\pi\le 3\pi$$

$$\Leftrightarrow -5<4k\le 3 \quad \Leftrightarrow \frac{-5}{4}<k\le\frac{3}{4} \Leftrightarrow \quad k=-1 \; ou \; k=0$$

$$\Leftrightarrow \quad x=\frac{\pi}{4}-\pi=\frac{-3\pi}{4} \quad ou \quad x=\frac{\pi}{4}$$

و أيضا:

تمرين 12

2	و أيضا: $\frac{-\pi}{12}+k\pi \in \left]-\pi;\pi\right] \Leftrightarrow -\pi < \frac{-\pi}{12}+k\pi \le \pi \Leftrightarrow -12\pi < -\pi+12k\pi \le 12\pi \Leftrightarrow -11\pi < 12k\pi \le 13\pi$ $\Leftrightarrow -11 < 12k \le 13 \Leftrightarrow \frac{-11}{12} < k \le \frac{13}{12} \Leftrightarrow k=0 \ ou \ k=1$ $\Leftrightarrow x=\frac{-\pi}{12} \ ou \ x=\frac{-\pi}{12}+\pi=\frac{11\pi}{12}$ بالتالي: $S=\left\{\frac{\pi}{4};\frac{-3\pi}{4};\frac{-\pi}{12};\frac{11\pi}{12}\right\}$

تمرين 13

1	لدينا: $A(x)=-2\cos^2(x)+\sqrt{3}\sin(2x)+2=2(1-\cos^2 x)+\sqrt{3}\times 2\sin x\cos x$ $=2\sin^2 x+2\sqrt{3}\sin x\cos x=4\sin x\left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)$ $=4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)$
2	$A(x)=0 \Leftrightarrow 4\sin(x)\left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)=0$ $\Leftrightarrow \sin(x)=0 \ ou \ \left(\frac{\sqrt{3}}{2}\cos(x)+\frac{1}{2}\sin(x)\right)=0$ $\Leftrightarrow x=k\pi / k\in Z \ ou \ \cos\left(\frac{\pi}{6}\right)\cos x+\sin\left(\frac{\pi}{6}\right)\sin x=0$ $\Leftrightarrow x=k\pi / k\in Z \ ou \ \cos\left(x-\frac{\pi}{6}\right)=0$ $\Leftrightarrow x=k\pi / k\in Z \ ou \ x-\frac{\pi}{6}=\frac{\pi}{2}+k\pi / k\in Z$ $\Leftrightarrow x=k\pi / k\in Z \ ou \ x=\frac{2\pi}{3}+k\pi / k\in Z$ بالتالي: $S=\{k\pi / k\in Z\}\cup\left\{\frac{2\pi}{3}+k\pi / k\in Z\right\}$