الامتحان الوطني الموحد للبكالوريا
المسالك الدولية
الدورة الاستدراكية 2021
- عناصر الإجابة -

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المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات

المادة	الرياضيات	مدة الإنجاز 4h
الشعبة أو المسلك	شعبة العلوم الرياضية (أ) و (ب) (خيار إنجليزية)	المعامل 9

Exercise 1 (7 points)			Elements of solutions	Marks
Part I	1-	a)	f is continuous on I	0.25
		b)	f is strictly decreasing I	0.25
		c)	$\lim\limits_{x\to 1^-} f(x)=-\infty$ …………………………………… $\lim\limits_{x\to -\infty} f(x)=+\infty$ …………………………………… $\lim\limits_{x\to -\infty} \frac{f(x)}{x}=0$ ……………………………………	0.25 0.25 0.25
		d)	The line of equation $x=1$ is a vertical asymptote to the curve (C)…………………………………… The x-axis is an asmptotic direction to the curve (C)……………………………………	0.25 0.25
		e)	Variation table of f.	0.25
	2-	a)	We have : $\forall x\in I \quad f''(x)=\frac{-1}{(x-1)^2}(<0)$, so the curve (C) is concave.	0.25
		b)	Representative curve (C).	0.25
	3-	a)	f is continuous and strictly decreasing over I *so* f *is a bijection from* I to $f(I)=\mathbb{R}$.	0.25
		b)	$\forall x\in\mathbb{R} \quad f^{-1}(x)=1-e^x$	0.25
		c)	The Verification.	0.25
Part II	1-		We apply the bijection's theorem to the function $x\mapsto P_n(x)-1$ on the interval $[0;1]$.	0.5
	2-		$P_2(x)=1\Leftrightarrow x^2+2x-2=0$, we find $\alpha=\sqrt{3}-1$ ……. $0<\sqrt{3}-1<1$ ……………………………………	0.25 0.25
	3-	a)	$P_{n+1}(x_n)=1+\frac{x_n^{n+1}}{n+1} \ (>1)$	0.5
		b)	For $n\geq 2$, we have : $P_{n+1}(x_n)>1\Rightarrow x_n>x_{n+1}$, so the sequence $(x_n)_{n\geq 2}$ is strictly decreasing .	0.5

		c)	The sequence $(x_n)_{n\geq 2}$ is strictly positive (by II-1-), in add it is strictly decreasing then its first term is an upper bound .	0.25
		d)	The sequence $(x_n)_{n\geq 2}$ is strictly decreasing and 0 is a lower bound , so it is convergent.	0.25
	4-	a)	$\forall x \in I \quad f'(x) = \dfrac{-1}{1-x}$ $\forall x \in I \quad P_n'(x) = 1 + x + x^2 + ... + x^{n-1} = \dfrac{1-x^n}{1-x}$ so $\forall x \in I \quad f_n'(x) = \dfrac{-x^n}{1-x}$	0.5
		b)	$\forall x \in [0,\alpha]; \forall n \geq 2 \quad \lvert f_n'(x)\rvert \leq \dfrac{\lvert x\rvert^n}{1-x} \leq \dfrac{\alpha^n}{1-x} \leq \dfrac{\alpha^n}{1-\alpha}$	0.25
		c)	We have : $\forall t \in [0,\alpha] \quad \lvert f_n'(t)\rvert \leq \dfrac{\alpha^n}{1-\alpha}$, so for $x \in [0,\alpha]$ we have by the means's inequality : $\left\lvert \int_0^x f_n'(t)dt \right\rvert \leq \dfrac{\alpha^n}{1-\alpha}.x$. Or $x \leq \alpha < 1$ so we obtain : $\lvert f_n(x)\rvert \leq \dfrac{\alpha^n}{1-\alpha}$ we can also use the MVT	0.5
		d)	We have : $x_n \in [0,\alpha]$ so $\lvert f_n(x_n)\rvert \leq \dfrac{\alpha^n}{1-\alpha}$ Hence $\lvert f(x_n)+P_n(x_n)\rvert \leq \dfrac{\alpha^n}{1-\alpha}$ or : $\lvert f(x_n)+1\rvert \leq \dfrac{\alpha^n}{1-\alpha}$	0.5
		e)	(by framing II-4-d). we have $\lim\limits_{n\to+\infty} \dfrac{\alpha^n}{1-\alpha} = 0$ ($0 < \alpha < 1$) so $\lim\limits_{n\to+\infty} f(x_n) = -1$. then : $\lim x_n = \lim f^{-1}(f(x_n)) = f^{-1}(-1) = 1 - e^{-1}$ (f^{-1} is continuous on $\mathbb{R}$).	0.5

Exercise 2		Elements of solutions	Marks
1-	a)	F is positive on $\mathbb{R}^+$ and negative on $\mathbb{R}^-$.	0.5
	b)	F is differentiable on $\mathbb{R}$ and we have : $\forall x \in \mathbb{R} \quad F'(x) = e^{x-\frac{x^2}{2}}$.	0.5 0.5
2-	a)	Integration by parts.	0.5
	b)	$\int_0^1 F(x)dx = \sqrt{e} - 1$	0.5

3-	a)	Verification.	0.5
	b)	We have : $\sum_{k=0}^{k=n-1}(n-k)F\left(\frac{k+1}{n}\right)=\sum_{k=1}^{k=n}(n-k+1)F\left(\frac{k}{n}\right)=\sum_{k=1}^{k=n}(n-k)F\left(\frac{k}{n}\right)+\sum_{k=1}^{k=n}F\left(\frac{k}{n}\right)$ we deduce the result : $u_n=\frac{1}{n}\sum_{k=1}^{k=n}F\left(\frac{k}{n}\right)-F(0)=\frac{1}{n}\sum_{k=1}^{k=n}F\left(\frac{k}{n}\right)$	0.5
	c)	The sequence $(u_n)_{n\geq 1}$ is convergent……………………………………… $\lim_{n\to+\infty} u_n=\int_0^1 F(x)dx=\sqrt{e}-1$ ………………………………………	0.25 0.25

Exercise3		Elements of solutions	Marks
1-	a)	Verification	0.5
	b)	$z_1=m$ et $z_2=-i$	0.5
	c)	Exponential form of z_1+z_2 in the case where $m=e^{i\frac{\pi}{8}}$	0.75
2-	a)	The affix of M' is $-\overline{m}$	0.5
	b)	The affix of N is $n=-\overline{m}+2+i$	0.75
	c)	Equivalence	1

Exercise 4		Elements of solutions	Marks
1-	a)	We have : $p/A \Rightarrow p/(a-1)A$ and $(a-1)A=a^7-1$ ……… Deduction : $\forall n\in\mathbb{N}\quad a^{7n}\equiv 1\ [p]$ ……………………	0.5 0.5
	b)	By Bezout's theorem ………………. Deduction : we use Fermat's theorem ………..	0.5 0.5
2-	a)	We have : $7 \nmid p-1$ then $7\wedge(p-1)=1$. By Bezout's theorem	0.5
	b)	$a\equiv 1[p]\Rightarrow A\equiv 7[p]$ $\Rightarrow p/7$ $\Rightarrow p=7$	0.5
3-		p is an odd prime number such that : p/A . We have two cases : if $7/p-1$ then $p\equiv 1\ [7]$ and if $7\nmid p-1$ then $p=7$	1